

**M R**  
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استاتيكا	فيزياء
الكترونيات	دوائر كهربية
هيدروليكا	ميكانيكا البناء

קורסخصائي

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• ملخص للمادة Pdf للمذكرة واطر اجعة

• محاضرات مباشرة على برنامج زووم

• طناشة الأجزاء الغير مفهومة

• تواصل مستمر مع فعلم اطادة

النواص

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## Structure Analysis 2

Principle of Virtual work  
Method of Virtual Work (Trusses)

### Lecture 2

# Method of Virtual Work: Trusses

## EXTERNAL LOADING

- To compute the vertical displacement  $\Delta$  of joint B of the truss caused by external loadings  $P_1$  and  $P_2$ :

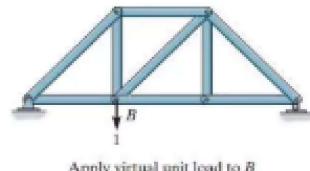
$$1 \cdot \Delta = \sum u \cdot dL$$

where  $dL = \frac{NL}{AE}$

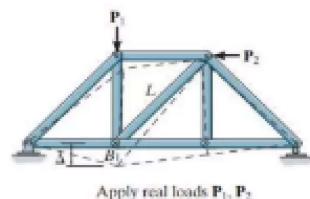
$$1 \cdot \Delta = \sum \frac{nNL}{AE}$$

where

$1$  = external virtual unit load acting on the truss joint in the stated direction of  $\Delta$



Apply virtual unit load to B  
(a)



Apply real loads  $P_1, P_2$   
(b)

$n$  = internal normal force in a truss member caused by the external virtual unit load

$\Delta$  = external joint displacement caused by the real loads on the truss

$N$  = internal normal force in a truss member caused by the real loads

$L$  = length of the member

$A$  = cross-sectional area of a member

$E$  = modulus of elasticity of a member

## TEMPERATURE

- In some cases, truss members may change their length due to temperature
  - If  $\alpha$  is the coefficient of thermal expansion for a member and  $\Delta T$  is the change in its temperature, the change in length of a member is
- $$\Delta L = \sum n\alpha \Delta T L$$
- To compute the displacement of a selected truss joint due to this temperature change

$$1 \cdot \Delta = \sum n\alpha \Delta T L$$

where

$1$  = external virtual unit load acting on the truss joint in the stated direction of  $\Delta$

$\alpha$  = coefficient of thermal expansion of a member

$\Delta T$  = change in temperature of member

## FABRICATION ERRORS AND CAMBERS

- Occasionally, errors in fabricating the lengths of the members of a truss may occur
- Also, in some cases truss members must be made slightly longer or shorter in order to give the truss a camber
- To compute the displacement of a truss joint from its expected position

$$1 \cdot \Delta = \sum n \Delta L$$

where

1 = external virtual unit load acting on the truss joint in the stated direction of  $\Delta$

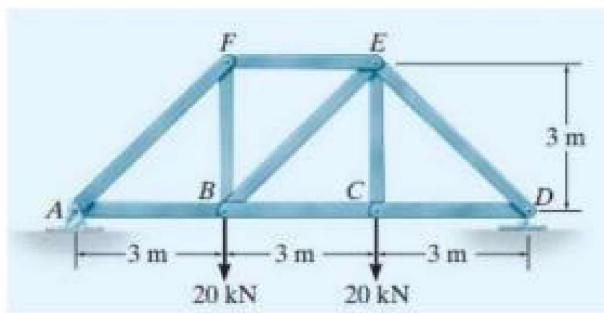
$\Delta L$  = difference in length of the member from its intended size as caused by a fabrication error

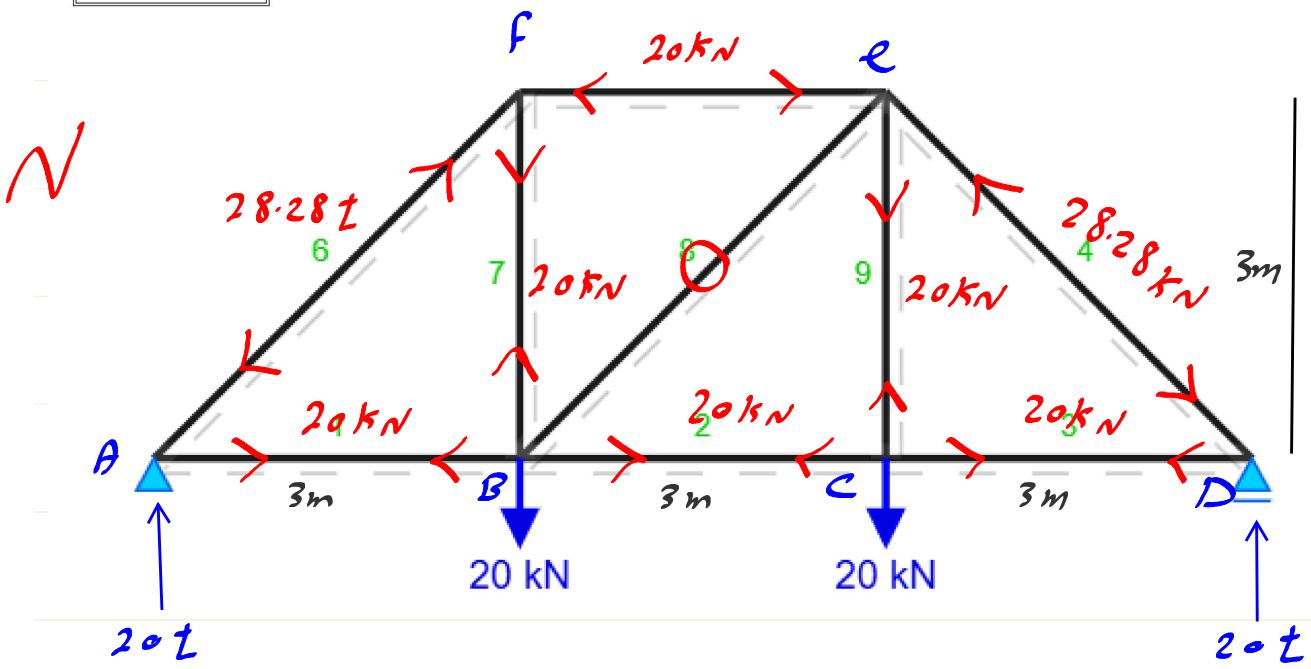
## EXAMPLE 1

### Problem:

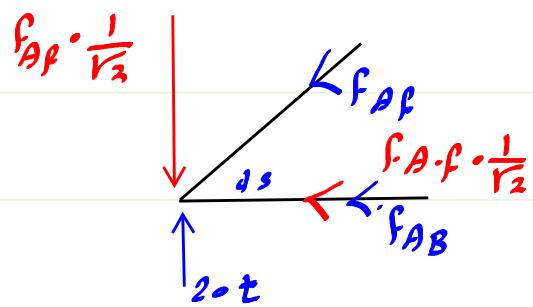
Determine the vertical displacement if joint C of the steel truss shown. The cross-sectional area of each member is  $A = 300 \text{ mm}^2$  and  $E = 200 \text{ Gpa}$ .

$$\Delta = \frac{\sum n \Delta L}{A E}$$





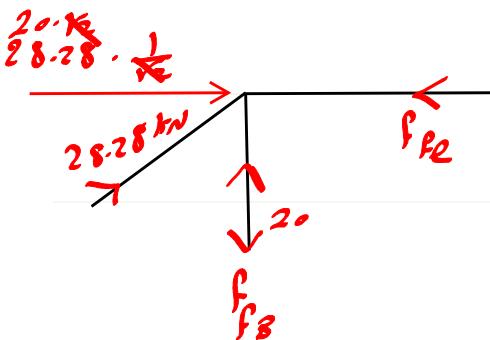
at joint A

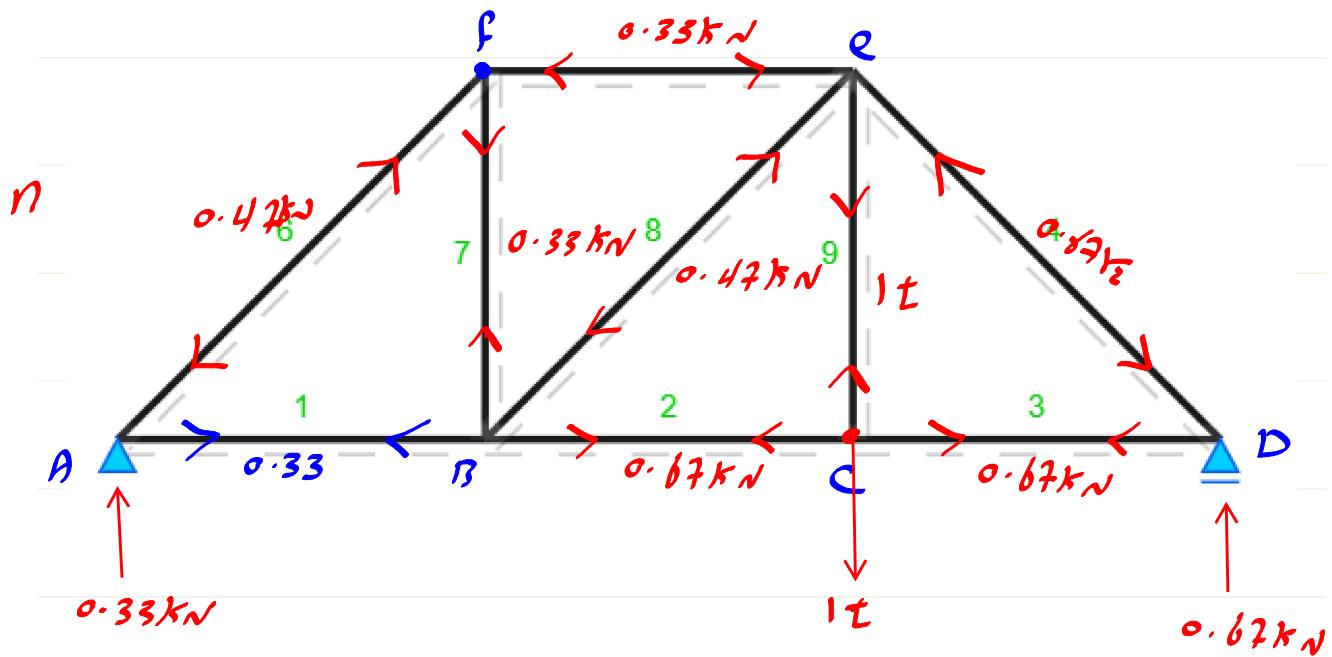


$$\sum f_y = 20 - f_{AB} \cdot \frac{1}{\sqrt{2}} = 0 \Rightarrow f_{AB} = 20\sqrt{2} = 28.28t$$

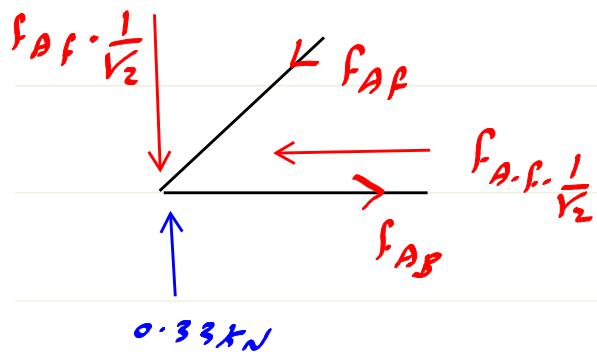
$$\sum f_x = -f_{AB} - f_{AF} \cdot \frac{1}{\sqrt{2}} = 0 \Rightarrow f_{AF} = -20t$$

at joint F



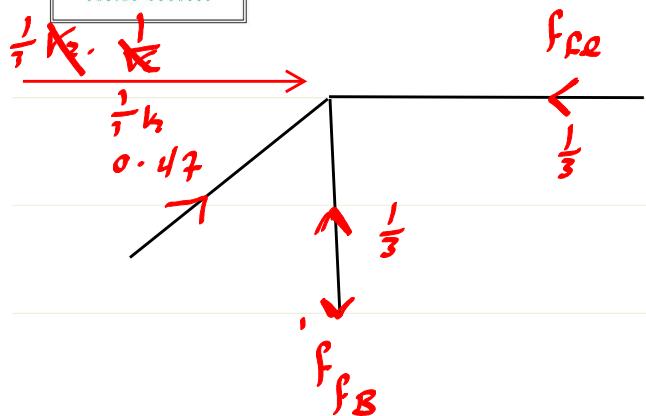


at joint A

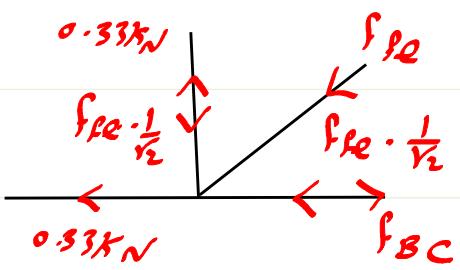


$$\sum F_y = 0.33 - f_{AF} \cdot \frac{1}{\sqrt{2}} = 0 \Rightarrow f_{AF} = 0.47 \text{ kN}$$

$$\sum F_x = f_{AB} - 0.47 \cdot \frac{1}{\sqrt{2}} = 0 \Rightarrow f_{AB} = 0.33 \text{ kN}$$



at point B



$$\sum F_y = 0.33 - f_{fe} \cdot \frac{1}{\sqrt{2}} = 0 \Rightarrow f_{fe} = 0.47 \text{ kN}$$

$$\sum F_x = -0.33 - 0.47 \cdot \frac{1}{\sqrt{2}} + f_{bc} = 0$$

H . V . D

member	n(KN)	N(KN)	L(m)	<u>nNL</u> (KN <sup>2</sup> .m)
AB	0.33	20	3	20
BC	0.67	20	3	40
CD	0.67	20	3	40
Ef	-0.33	-20	5	20
FB	0.33	20	3	20
EC	1	20	3	60
AF	-0.17	-28.28	4.24	56.56
BE	-0.47	0	4.24	0
ED	-0.947	-28.28	4.24	115.55

$\sum nNL$

369.91

$$A = 300 \text{ mm}^2 = 300 \times (10^{-3})^2 \text{ m}^2$$

$$E = 200 \text{ GPa} = 200 \times 10^9 \text{ Pa}$$

$$\Delta = \frac{\sum nNL}{AE} = \frac{369.91 \times 10^3}{300 \times 10^{-6} \times 200 \times 10^9} = 6.16 \times 10^{-3} \text{ m}$$

$$= 6.16 \text{ mm}$$