Structural Analysis II

Influence Line

Outlines

Outlines

Outlines

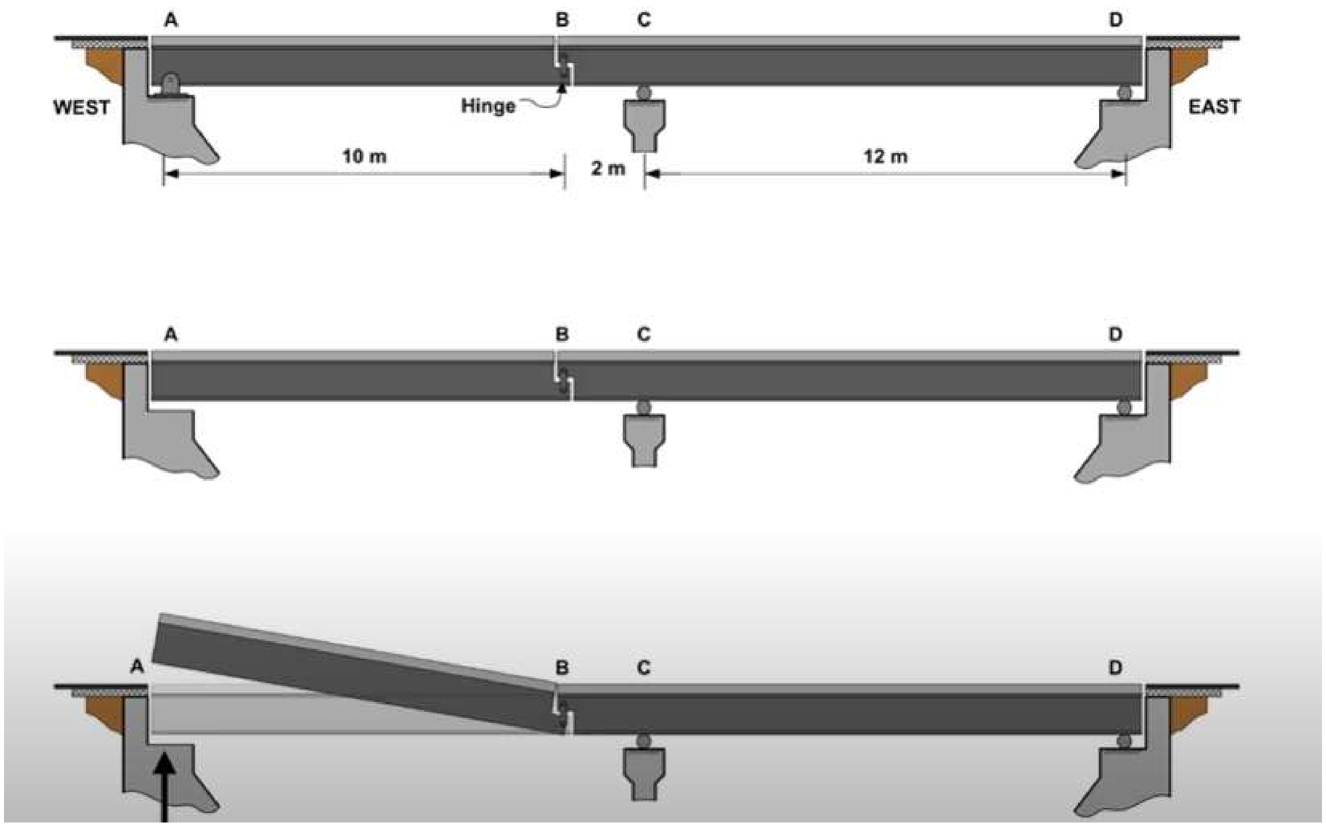
Support Connections

- An **influence line** represents the variation of either the reaction, shear, or moment, at a *specific point* in a member as a concentrated or distributed load moves over the member.

- If a structure is subjected to a *live* or *moving load*, however, the variation of the shear and bending moment in the member is best described using the ***influence line***.

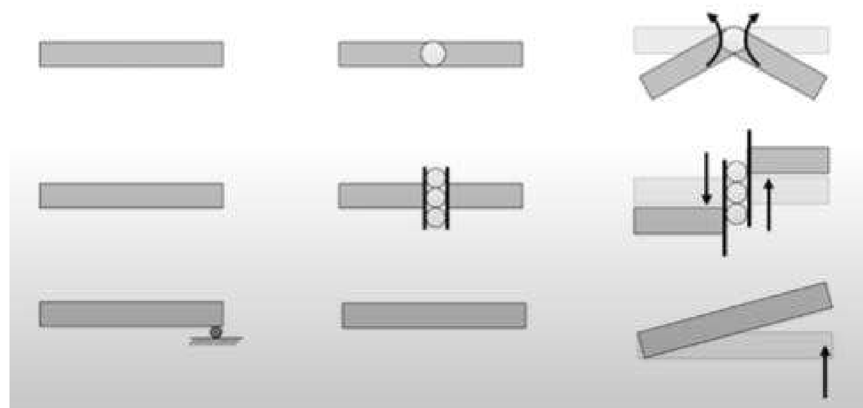
Support Connections

- For these reasons, influence lines play an important part in the design of bridges, industrial crane rails, conveyors, and other structures where loads move across their span.


A
B
C
D
WEST
EAST
Hinge
10 m
2 m
12 m

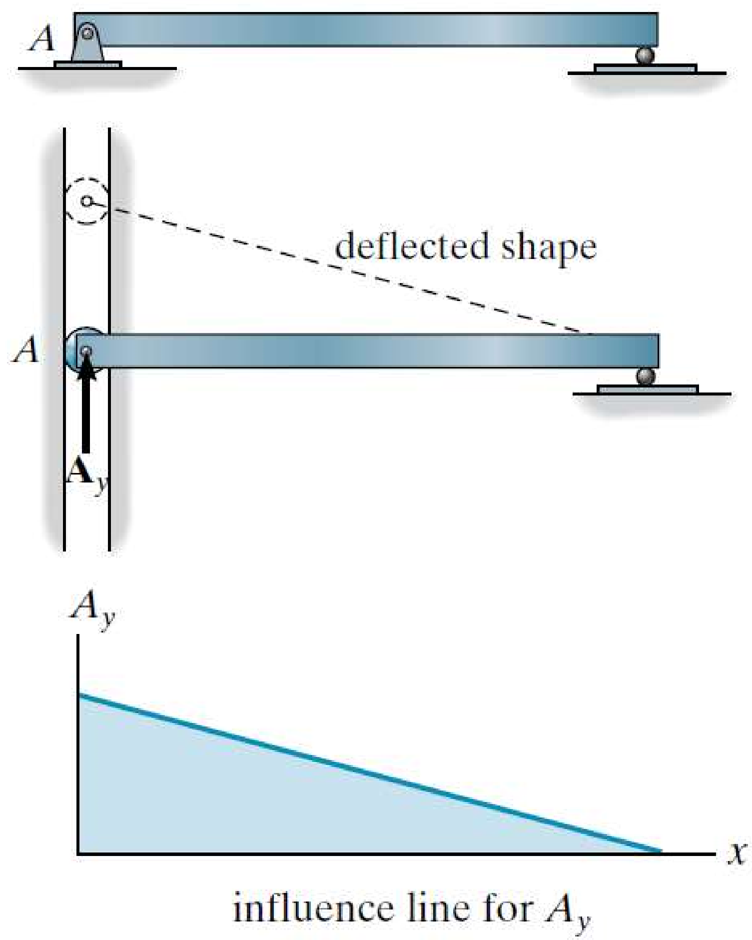
Qualitative Influence Lines

- In 1886, Heinrich Müller-Breslau developed a technique for rapidly constructing the shape of an influence line. Referred to as the ***Müller-Breslau principle.***
- In order to draw the deflected shape properly, the capacity of the beam to resist the applied function must be *removed* so the beam can deflect when the function is applied.
- ***Moment***
- ***Shear***
- ***Reactions***



Qualitative Influence Lines

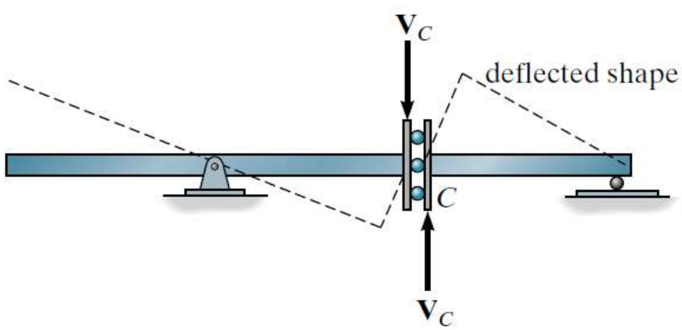
- Vertical Reactions



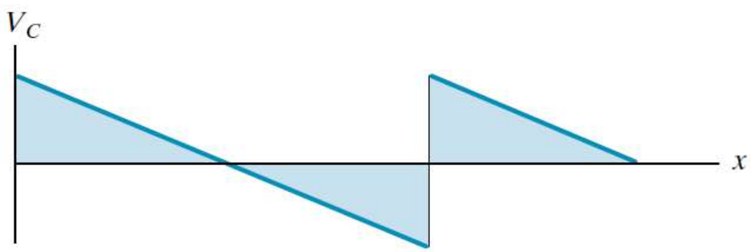
Qualitative Influence Lines

Shear

(a)

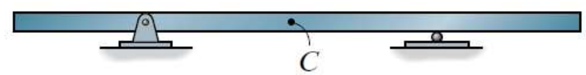


(b)

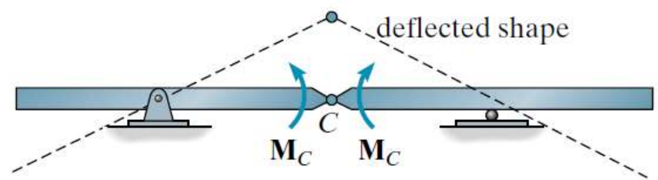


influence line for V_C

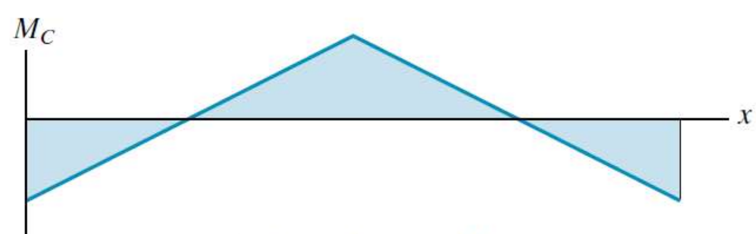
Moment



(a)



(b)



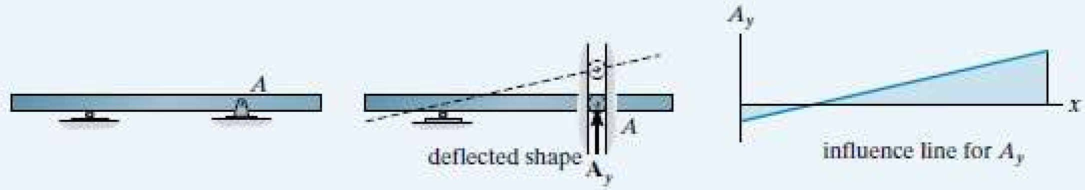
influence line for M_C

EXAMPLE 6.9

For each beam in Fig. 6–16*a* through 6–16*c*, sketch the influence line for the vertical reaction at *A*.

SOLUTION

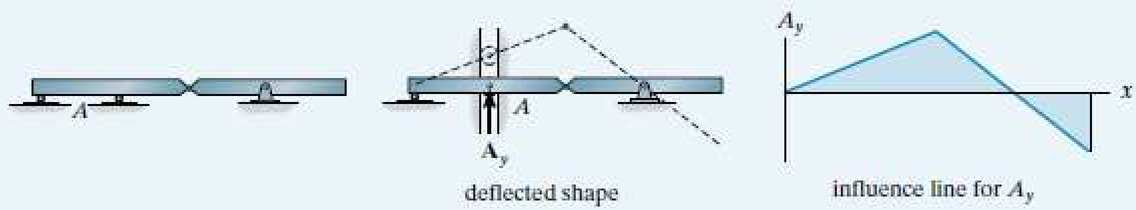
The support is replaced by a roller guide at *A* since it will resist $\mathbf{A}_x$, but not $\mathbf{A}_y$. The force $\mathbf{A}_y$ is then applied.



(a)

Fig. 6–16

Again, a roller guide is placed at *A* and the force $\mathbf{A}_y$ is applied.



(b)

A *double-roller guide* must be used at *A* in this case, since this type of support will resist both a moment $\mathbf{M}_A$ at the fixed support and axial load $\mathbf{A}_x$, but will not resist $\mathbf{A}_y$.

EXAMPLE 6.10

For each beam in Figs. 6–17*a* through 6–17*c*, sketch the influence line for the shear at *B*.

SOLUTION

The roller guide is introduced at *B* and the positive shear $\mathbf{V}_B$ is applied. Notice that the right segment of the beam will *not deflect* since the roller at *A* actually constrains the beam from moving vertically, either up or down. [See support (2) in Table 2.1.]

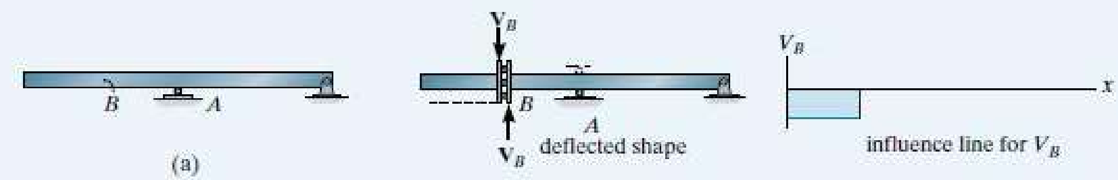
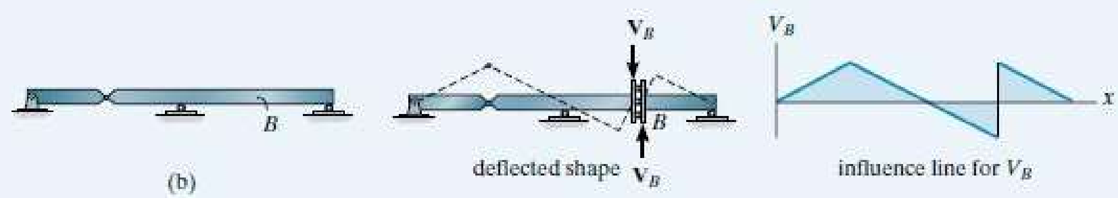
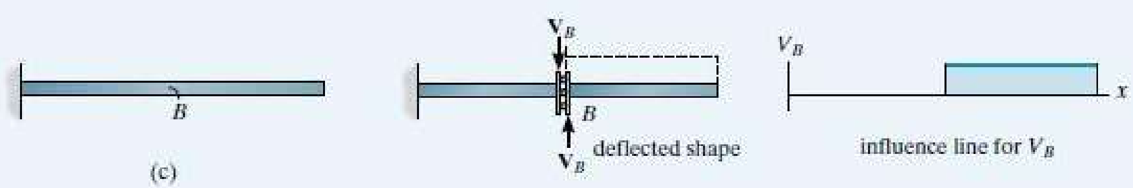


Fig. 6–17

Placing the roller guide at *B* and applying the positive shear at *B* yields the deflected shape and corresponding influence line.



Again, the roller guide is placed at *B*, the positive shear is applied, and the deflected shape and corresponding influence line are shown. Note that the left segment of the beam does not deflect, due to the fixed support.



EXAMPLE 6.11

For each beam in Figs. 6–18*a* through Fig. 6–18*c*, sketch the influence line for the moment at B.

SOLUTION

A hinge is introduced at *B* and positive moments $\mathbf{M}_B$ are applied to the beam. The deflected shape and corresponding influence line are shown.

(a)

Fig. 6–18

Placing a hinge at *B* and applying positive moments $\mathbf{M}_B$ to the beam yields the deflected shape and influence line.

(b)

With the hinge and positive moment at *B* the deflected shape and influence line are shown. The left segment of the beam is constrained from moving due to the fixed wall at *A*.

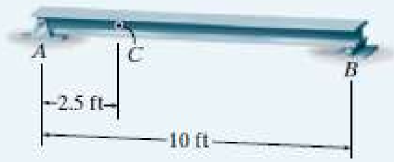
(c)

Calculating Influence Line

- 1. Place Unit Load (1) at specific location
- 2. Calculate the reactions
- 3. Draw the influence line and calculate influence line ordinate
- 4. Calculating the maximum reactions or shear or moment

EXAMPLE 6.7

Determine the maximum *positive* shear that can be developed at point C in the beam shown in Fig. 6–10*a* due to a concentrated moving load of 4000 lb and a uniform moving load of 2000 lb/ft.



(a)

Fig. 6–10

influence line for V_C

(b)

SOLUTION

The influence line for the shear at C has been established in Example 6.3 and is shown in Fig. 6–10*b*.

Concentrated Force. The maximum positive shear at C will occur when the 4000-lb force is located at $x = 2.5^+$ ft, since this is the positive peak of the influence line. The ordinate of this peak is +0.75; so that

$$V_C = 0.75(4000 \text{ lb}) = 3000 \text{ lb}$$

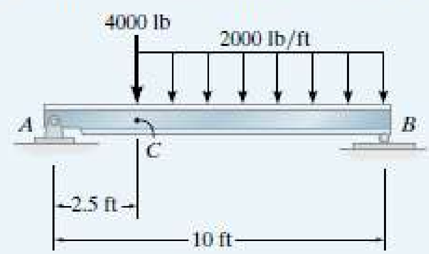
Uniform Load. The uniform moving load creates the maximum positive influence for V_C when the load acts on the beam between $x = 2.5^+$ ft and $x = 10$ ft, since within this region the influence line has a positive area. The magnitude of $\mathbf{V}_C$ due to this loading is

$$V_C = \left[\tfrac{1}{2}(10 \text{ ft} - 2.5 \text{ ft})(0.75)\right]2000 \text{ lb/ft} = 5625 \text{ lb}$$

Total Maximum Shear at C.

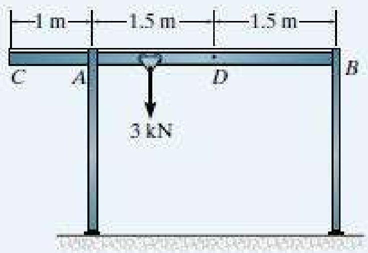
$$(V_C)_{max} = 3000 \text{ lb} + 5625 \text{ lb} = 8625 \text{ lb} \qquad \textit{Ans.}$$

Notice that once the *positions* of the loads have been established using the influence line, Fig. 6–10*c*, this value of $(V_C)_{max}$ can *also* be determined using statics and the method of sections. Show that this is the case.

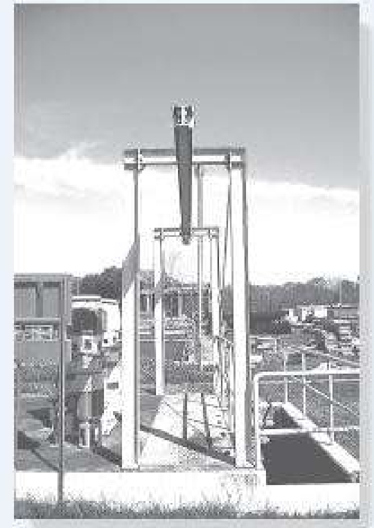


EXAMPLE 6.8

The frame structure shown in Fig. 6–11*a* is used to support a hoist for transferring loads for storage at points underneath it. It is anticipated that the load on the dolly is 3 kN and the beam *CB* has a mass of 24 kg/m. 9.81 Assume the dolly has negligible size and can travel the entire length of the beam. Also, assume *A* is a pin and *B* is a roller. Determine the maximum vertical support reactions at *A* and *B* and the maximum moment in the beam at *D*.



(a)



SOLUTION

Maximum Reaction at A. We first draw the influence line for A_y, Fig. 6–11*b*. Specifically, when a unit load is at *A* the reaction at *A* is 1 as shown. The ordinate at *C*, is 1.33. Here the maximum value for A_y occurs when the dolly is at *C*. Since the dead load (beam weight) must be placed over the entire length of the beam, we have,

$$(A_y)_{\max} = 3000(1.33) + 24(9.81)\left[\tfrac{1}{2}(4)(1.33)\right]$$
$$= 4.63 \text{ kN} \qquad \textit{Ans.}$$

Maximum Reaction at B. The influence line (or beam) takes the shape shown in Fig. 6–11*c*. The values at *C* and *B* are determined by statics. Here the dolly must be at *B*. Thus,

$$(B_y)_{\max} = 3000(1) + 24(9.81)\left[\tfrac{1}{2}(3)(1)\right] + 24(9.81)\left[\tfrac{1}{2}(1)(-0.333)\right]$$
$$= 3.31 \text{ kN} \qquad \textit{Ans.}$$

Maximum Moment at D. The influence line has the shape shown in Fig. 6–11*d*. The values at *C* and *D* are determined from statics. Here,

$$(M_D)_{\max} = 3000(0.75) + 24(9.81)\left[\tfrac{1}{2}(1)(-0.5)\right] + 24(9.81)\left[\tfrac{1}{2}(3)(0.75)\right]$$
$$= 2.46 \text{ kN}\cdot\text{m} \qquad \textit{Ans.}$$

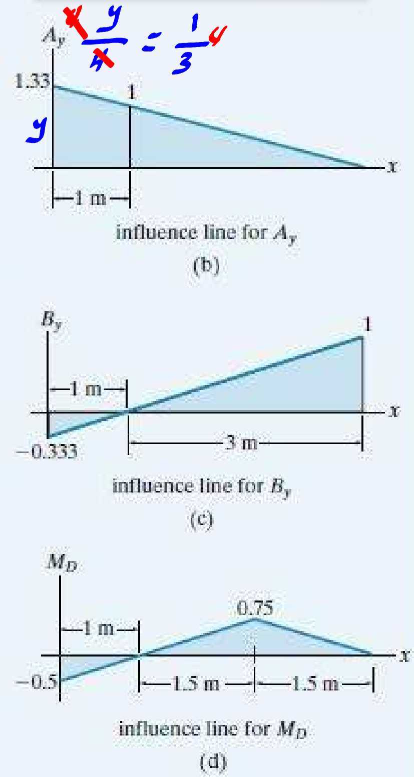
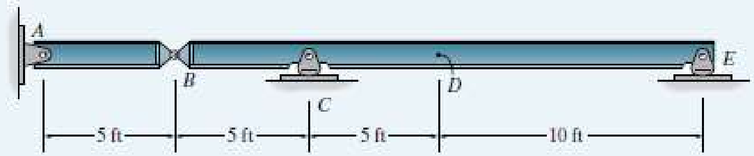


Fig. 6–11

EXAMPLE 6.12

Determine the maximum positive moment that can be developed at point D in the beam shown in Fig. 6–19a due to a concentrated moving load of 4000 lb, a uniform moving load of 300 lb/ft, and a beam weight of 200 lb/ft.



(a)

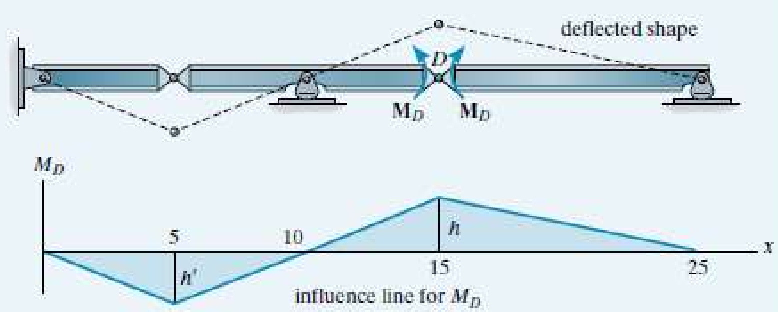
Fig. 6–19

SOLUTION

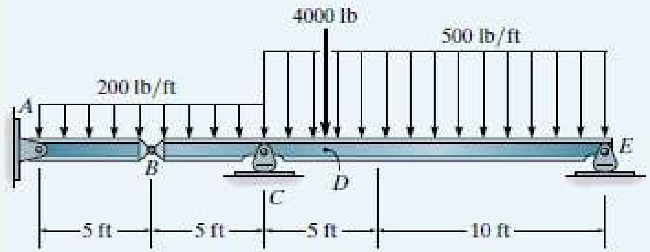
A hinge is placed at D and positive moments $\mathbf{M}_D$ are applied to the beam. The deflected shape and corresponding influence line are shown in Fig. 6–19b. Immediately one recognizes that the concentrated moving load of 4000 lb creates a maximum *positive* moment at D when it is placed at D, i.e., the peak of the influence line. Also, the uniform moving load of 300 lb/ft must extend from C to E in order to cover the region where the area of the influence line is positive. Finally, the uniform *dead weight* of 200 lb/ft acts over the *entire length* of the beam. The loading is shown on the beam in Fig. 6–19c. Knowing the position of the loads, we can now determine the maximum moment at D using statics. In Fig. 6–19d the reactions on BE have been calculated. Sectioning the beam at D and using segment DE, Fig. 6–19e, we have

$$\circlearrowleft +\Sigma M_D = 0; \qquad -M_D - 5000(5) + 4750(10) = 0$$

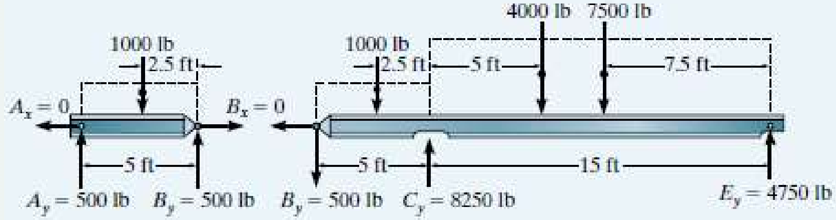
$$M_D = 22\,500 \text{ lb}\cdot\text{ft} = 22.5 \text{ k}\cdot\text{ft} \qquad \textit{Ans.}$$



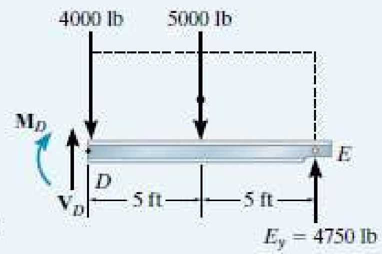
(b)



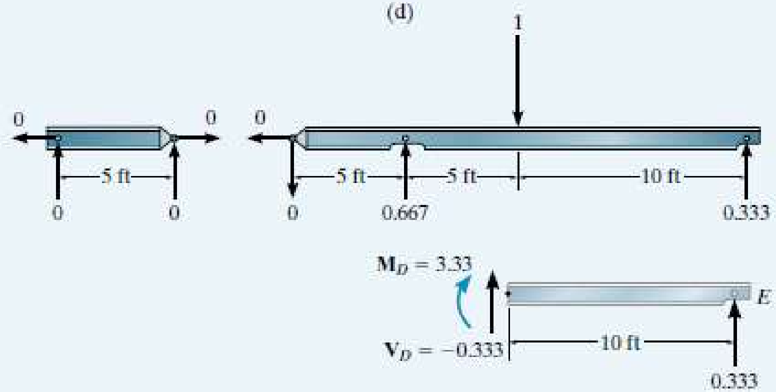
(c)



(d)



(e)



(f)

This problem can also be worked by using *numerical values* for the influence line as in Sec. 6.1. Actually, by inspection of Fig. 6–19b, only the peak value h at D must be determined. This requires placing a unit load on the beam at D in Fig. 6–19a and then solving for the internal moment in the beam at D. The calculations are shown in Fig. 6–19f. Thus $M_D = h = 3.33$. By proportional triangles, $h'/(10 - 5) = 3.33/(15 - 10)$ or $h' = 3.33$. Hence, with the loading on the beam as in Fig. 6–19c, using the areas and peak values of the influence line, Fig. 6–19b, we have

$$M_D = 500\left[\frac{1}{2}(25 - 10)(3.33)\right] + 4000(3.33) - 200\left[\frac{1}{2}(10)(3.33)\right]$$

$$= 22\,500 \text{ lb} \cdot \text{ft} = 22.5 \text{ k} \cdot \text{ft} \qquad \textit{Ans.}$$

Example 9.3

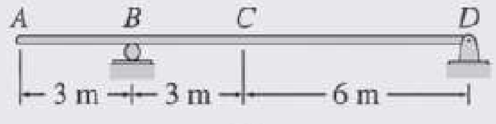
For the beam shown in Fig. 9.5(a), determine the maximum positive and negative shears and the maximum positive and negative bending moments at point C due to a concentrated live load of 90 kN, a uniformly distributed live load of 40 kN/m, and a uniformly distributed dead load of 20 kN/m.

Solution

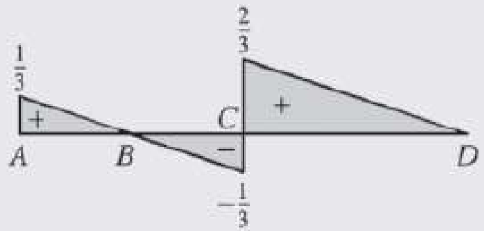
Influence Lines. The influence lines for the shear and bending moment at point C of this beam were previously constructed in Example 8.6 and are shown in Fig. 9.5(b) and (e), respectively.

Maximum Positive Shear at C. To obtain the maximum positive shear at C due to the 90-kN concentrated live load, we place the load just to the right of C (Fig. 9.5(c)), where the maximum positive ordinate (2/3 kN/kN) of the influence line

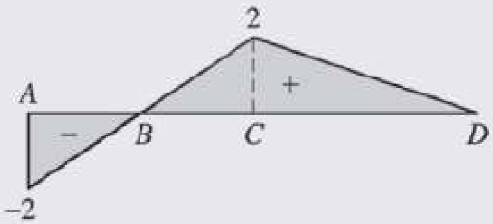
continued



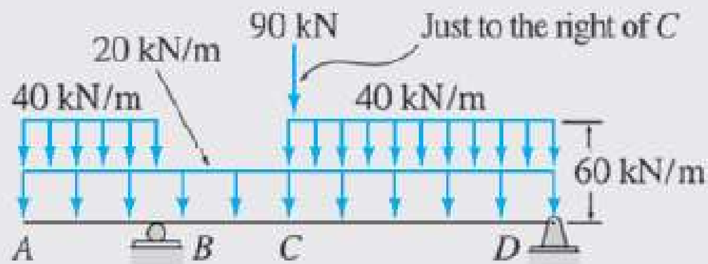
(a)



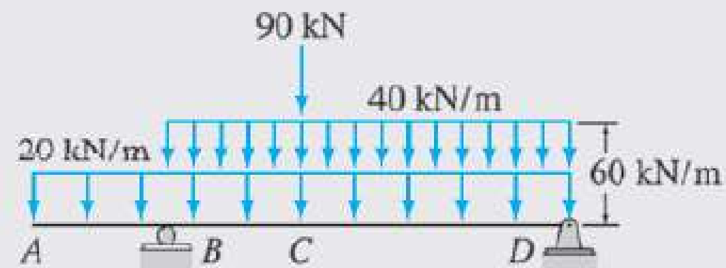
(b) Influence Line for S_C (kN/kN)



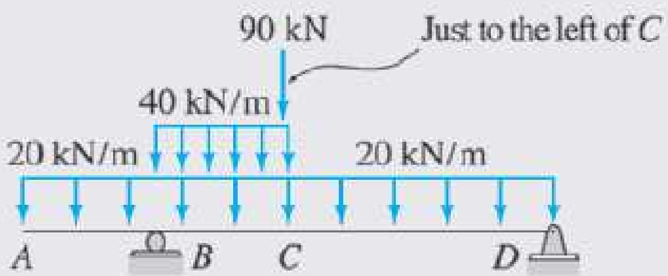
(e) Influence Line for M_C (kN·m/kN)



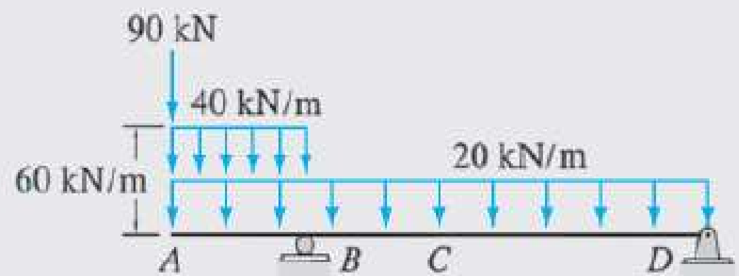
(c) Loading Arrangement for Maximum Positive S_C



(f) Loading Arrangement for Maximum Positive M_C



(d) Loading Arrangement for Maximum Negative S_C



(g) Loading Arrangement for Maximum Negative M_C

for S_C occurs. By multiplying the magnitude of the load by the value of this ordinate, we determine the maximum positive value of S_C due to the concentrated live load as

$$S_C = 90\left(\frac{2}{3}\right) = 60 \text{ kN}$$

From Fig. 9.5(b), we observe that the ordinates of the influence line for S_C are positive between the points A and B and between the points C and D. Therefore, to obtain the maximum positive shear at C due to the 40-kN/m uniformly distributed live load, we place the load over the portions AB and CD of the beam, as shown in Fig. 9.5(c), and compute the maximum positive value of S_C due to this load by multiplying the load intensity by the area under the portions AB and CD of the influence line. Thus

$$S_C = 40\left[\left(\frac{1}{2}\right)(3)\left(\frac{1}{3}\right) + \left(\frac{1}{2}\right)(6)\left(\frac{2}{3}\right)\right] = 100 \text{ kN}$$

Unlike live loads, the dead loads always act at fixed positions on structures; that is, their positions cannot be varied to maximize response functions. Therefore, the 20-kN/m uniformly distributed dead load is placed over the entire length of the beam, as shown in Fig. 9.5(c), and the corresponding shear at C is determined by multiplying the dead-load intensity by the net area under the entire influence line as

$$S_C = 20\left[\left(\frac{1}{2}\right)(3)\left(\frac{1}{3}\right) + \left(\frac{1}{2}\right)(3)\left(-\frac{1}{3}\right) + \left(\frac{1}{2}\right)(6)\left(\frac{2}{3}\right)\right] = 40 \text{ kN}$$

The total maximum positive shear at C can now be obtained by algebraically adding the values of S_C determined for the three types of loads.

$$\text{Maximum positive } S_C = 60 + 100 + 40 = 200 \text{ kN} \qquad \textbf{Ans.}$$

Maximum Negative Shear at C. The arrangement of the loads to obtain the maximum negative shear at C is shown in Fig. 9.5(d). The maximum negative shear at C is given by

$$\begin{aligned}\text{Maximum negative } S_C &= 90\left(-\frac{1}{3}\right) + 40\left(\frac{1}{2}\right)(3)\left(-\frac{1}{3}\right) + 20\left[\left(\frac{1}{2}\right)(3)\left(\frac{1}{3}\right)\right.\\ &\quad \left. + \left(\frac{1}{2}\right)(3)\left(-\frac{1}{3}\right) + \left(\frac{1}{2}\right)(6)\left(\frac{2}{3}\right)\right]\\ &= -10 \text{ kN}\end{aligned} \qquad \textbf{Ans.}$$

Maximum Positive Bending Moment at *C*. The arrangement of the loads to obtain the maximum positive bending moment at C is shown in Fig. 9.5(f). Note that the 90-kN concentrated live load is placed at the location of the maximum positive ordinate of the influence line for M_C (Fig. 9.5(e)); the 40-kN/m uniformly distributed live load is placed over the portion BD of the beam, where the ordinates of the influence line are positive; whereas the 20-kN/m uniformly distributed dead load is placed over the entire length of the beam. The maximum positive bending moment at C is given by

$$\begin{aligned}\text{Maximum positive } M_C &= 90(2) + 40\left(\frac{1}{2}\right)(9)(2) \\ &\quad + 20\left[\left(\frac{1}{2}\right)(3)(-2) + \left(\frac{1}{2}\right)(9)(2)\right] \\ &= 660 \text{ kN} \cdot \text{m}\end{aligned}$$

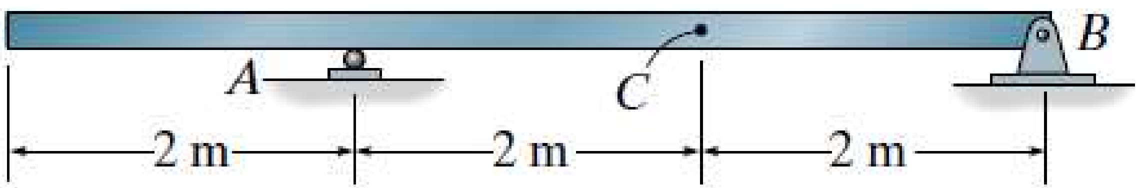
Ans.

Maximum Negative Bending Moment at *C*. The loading arrangement to obtain the maximum negative bending moment at C is shown in Fig. 9.5(g). The maximum negative M_C is given by

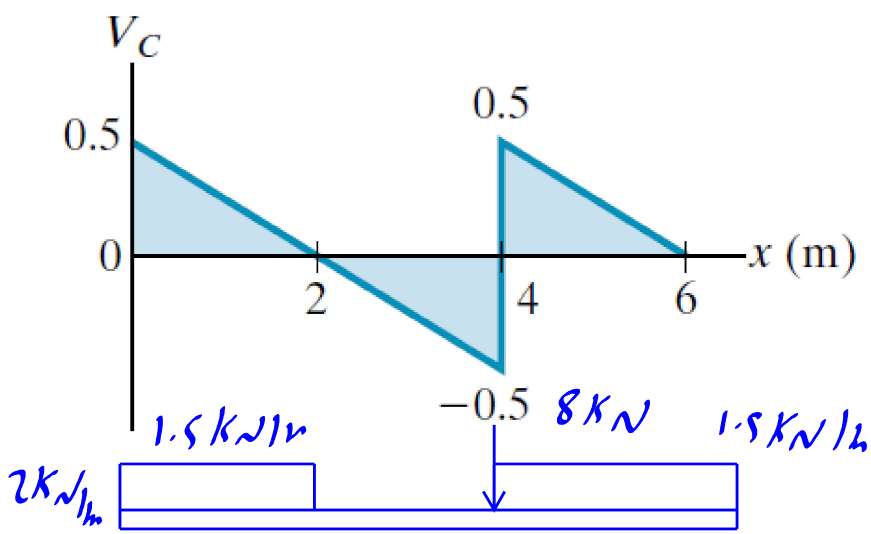
$$\begin{aligned}\text{Maximum negative } M_C &= 90(-2) + 40\left(\frac{1}{2}\right)(3)(-2) \\ &\quad + 20\left[\left(\frac{1}{2}\right)(3)(-2) + \left(\frac{1}{2}\right)(9)(2)\right] \\ &= -180 \text{ kN} \cdot \text{m}\end{aligned}$$

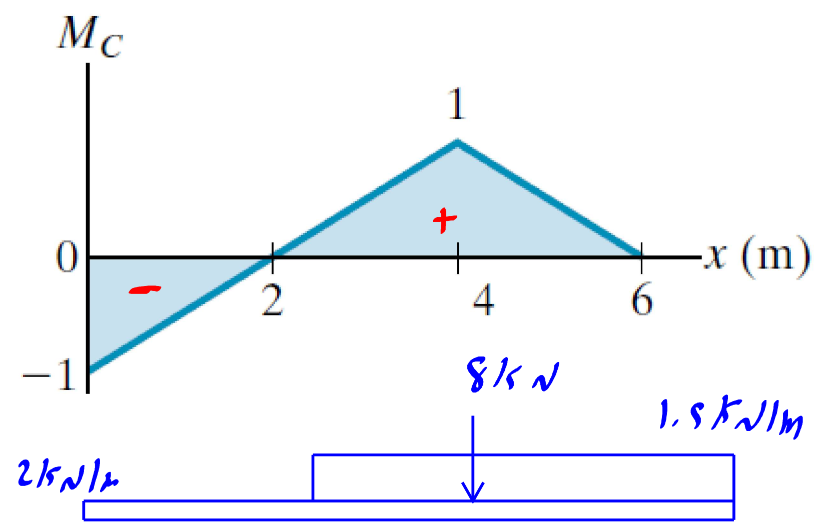
Ans.

F6–7. The beam supports a distributed live load of 1.5 kN/m and single concentrated load of 8 kN. The dead load is 2 kN/m. Determine (a) the maximum positive moment at C, (b) the maximum positive shear at C.



F6–7





$$
\begin{aligned}
(M_C)_{\max(+)} &= 8(1) + \left[\frac{1}{2}(6 - 2)(1)\right](1.5) \\
&\quad + \left[\frac{1}{2}(2)(-1)\right](2) + \left[\frac{1}{2}(6 - 2)(1)\right](2) \\
&= 13.0\ \text{kN}\cdot\text{m} \qquad \textit{Ans.} \\
(V_C)_{\max(+)} &= 8(0.5) + \left[\frac{1}{2}(2)(0.5)\right](1.5) + \left[\frac{1}{2}(6 - 4)(0.5)\right](1.5) \\
&\quad + \left[\frac{1}{2}(2)(0.5)\right](2) + \left[\frac{1}{2}(4 - 2)(-0.5)\right](2) + \\
&\quad \left[\frac{1}{2}(6 - 4)(0.5)\right](2) \\
&= 6.50\ \text{kN} \qquad \textit{Ans.}
\end{aligned}
$$