
AHMEDMAHDY
MR
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مدرس خصوصى
اونلاين
يحصل الطالب على ...
*مقاطع فيديوهات لشرح المقرربشكل وافي
*ملخص للمادة للمراجعة النهائية
*محاضرات مباشرة على برنامج Zoom
*تواصل مستمر مع معلم المادة للمناقشة
للتواصل
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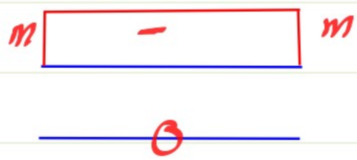
00201096019763

(1)



B.M.D

S.F.D

/ boundary conditions

at $x = 0 \Rightarrow \theta = 0 \,,\, \Delta = 0$

$$\theta = \frac{1}{EI}\int M dx = -\frac{M}{EI}x + C_1$$

$$/\ \Delta = \frac{1}{EI}\iint M dx = \frac{1}{EI}\int(-Mx + C_1)\,dx$$

$$= \frac{1}{EI}\left[-M\frac{x^2}{2} + C_1 x + C_2\right]$$

/ at $x = 0$

$$\theta = -\frac{M}{EI}(0) + C_1 = 0 \Rightarrow C_1 = 0$$

$$\theta = \frac{-M}{EI}x \Rightarrow \theta_B = \frac{-M}{EI}L$$

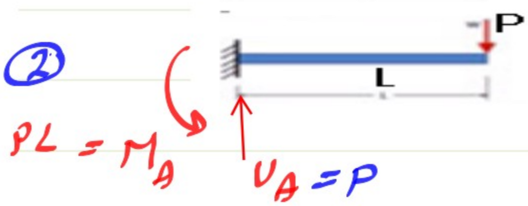
/

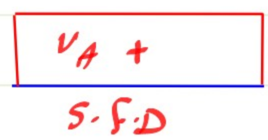
$$\Delta = \frac{1}{EI}\left[-M\left(\frac{0}{2}\right) + 0 + C_2\right] = 0 \Rightarrow C_2 = 0$$

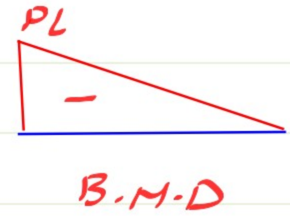
$$\Delta = -M\frac{x^2}{2} \,,\ \max \Delta = -\frac{ML^2}{2}$$

Max value of $M = M$

(2)







$$M = V_A x - M_A = Px - PL$$

$$\theta = \frac{1}{EI}\int M\,dx = \frac{1}{EI}\int (Px - PL)\,dx$$

$$\theta = \frac{1}{EI}\left(P\frac{x^2}{2} - PLx + C_1\right)$$

$$\Delta = \frac{1}{EI}\int \theta\,dx = \frac{1}{EI}\int \left(P\frac{x^2}{2} - PLx + C_1\right)dx$$

$$\Delta = \frac{1}{EI}\left[P\frac{x^3}{6} - PL\frac{x^2}{2} + C_1 x + C_2\right]$$

at $x = 0 \Rightarrow \theta = 0,\ \Delta = 0$

$$\theta = P(0) - PL(0) + C_1 = 0 \Rightarrow C_1 = 0$$

$$\Delta = P(0) - PL(0) + C_1(0) + C_2 = 0 \Rightarrow C_2 = 0$$

$$\theta = P\frac{x^2}{2} - PLx \Rightarrow \theta_B = P\frac{L^2}{2} - PL^2 = -\frac{PL^2}{2EI}$$

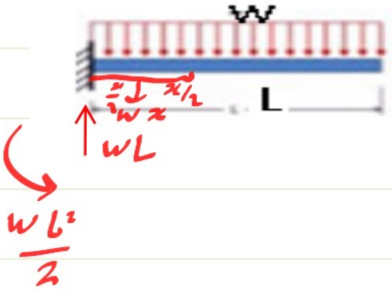
$$\Delta = \frac{1}{EI}\left(P\frac{x^3}{6} - PL\frac{x^2}{2}\right]$$

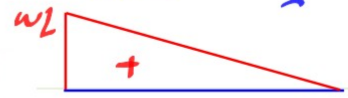
$$\Delta_B = \frac{1}{EI}\left(\frac{PL^3}{6} - \frac{PL^3}{2}\right) = -\frac{PL^3}{3EI}$$

max value of B.M = PL

$$M = -\frac{wL^2}{2} + wLx - \frac{wx^2}{2}$$

③





S.F.D

B.M.D

$$\theta = \frac{1}{EI}\int M\,dx = \frac{1}{EI}\int\left(-\frac{wL^2}{2} + wLx - \frac{wx^2}{2}\right)dx$$

$$\theta = \frac{1}{EI}\left(-\frac{wL^2}{2}x + wL\frac{x^2}{2} - \frac{wx^3}{6} + C_1\right)$$

$$\Delta = \frac{1}{EI}\int\theta\,dx = \frac{1}{EI}\left[-\frac{wL^2}{2}\cdot\frac{x^2}{2} + wL\frac{x^3}{6} - \frac{wx^4}{24} + C_1x + C_2\right.$$

at $x = 0 \Rightarrow \theta = 0,\ \Delta = 0 \Rightarrow C_1 = C_2 = 0$

$$\theta = \frac{1}{EI}\left(-\frac{wL^2}{2}x + wL\frac{x^2}{2} - \frac{wx^3}{6}\right)$$

$$\theta_B = \frac{1}{EI}\left(\frac{-wL^3}{2} + w\frac{L^3}{2} - \frac{wL^3}{6}\right) = -\frac{wL^3}{6EI}$$

$$\Delta = \frac{-wL^2x^2}{4} + \frac{wLx^3}{6} - \frac{wx^4}{24}$$

$$\Delta_B = \frac{-wL^4}{4} + \frac{wL^4}{6} - \frac{wL^4}{24} = -\frac{wL^4}{8EI}$$

Max value of B.M $= \frac{wL^2}{2}$

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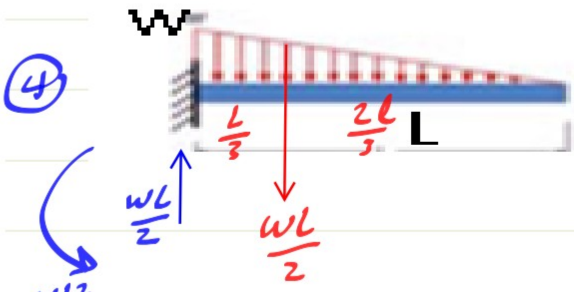

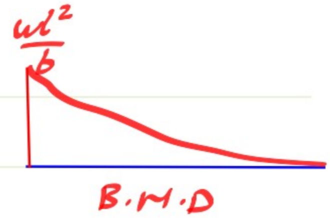

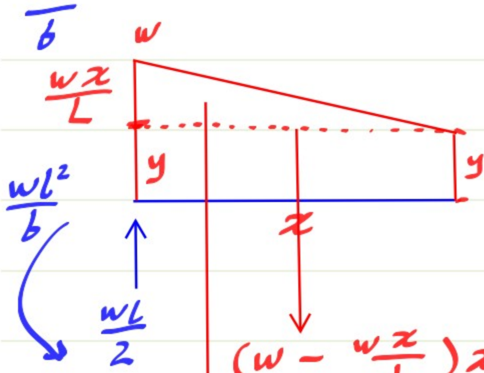

$$y = w - \frac{wx}{L}$$

$$\left(w - \frac{wx}{L}\right)x = wx - \frac{wx^2}{L}$$

$$\frac{wx}{L}\,\frac{x}{2} = \frac{wx^2}{2L}$$

$$M = -\frac{wL^2}{6} + \frac{wL}{2}x - \frac{wx^2}{2L}\cdot\frac{2}{3}x - \left(wx - \frac{wx^2}{L}\right)\left(\frac{x}{2}\right)$$

$$M = -\frac{wL^2}{6} + \frac{wL}{2}x - \frac{wx^3}{3L} - \frac{wx^2}{2} + \frac{wx^3}{2L}$$

$$\theta = \frac{1}{EI}\left\{-\frac{wL^2}{6}x + \frac{wL}{2}\cdot\frac{x^2}{2} - \frac{w}{3L}\cdot\frac{x^4}{4} - \frac{w}{2}\cdot\frac{x^3}{3} + \frac{w}{2L}\cdot\frac{x^4}{4}\right\} + C_1$$

$$\Delta = -\frac{wL^2}{6}\cdot\frac{x^2}{2} + \frac{wL}{4}\cdot\frac{x^3}{3} - \frac{w}{12L}\cdot\frac{x^5}{5} - \frac{w}{6}\,\frac{x^4}{4} + \frac{w}{8L}\,\frac{x^5}{5} + C_1x + C_2$$

at $x = 0 \Rightarrow \Delta = 0$, $\theta = 0 \Rightarrow C_1 = C_2 = 0$

$$\theta_B = -\frac{wL^3}{24}$$

$\Delta_B = \frac{-wL^4}{30}$, Max Binding $= \frac{wL^2}{6}$

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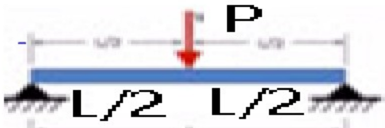
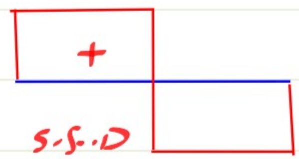
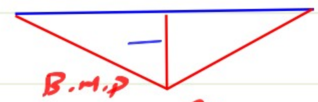
$$M = \frac{P}{2}x$$

3







$$\theta = \frac{1}{EI}\int M\,dx = \frac{1}{EI}\left(\frac{Px^2}{4} + C_1\right)$$

$$,\ \Delta = \frac{1}{EI}\int \theta\,dx = \frac{1}{EI}\left[\frac{Px^3}{12} + C_1 x + C_2\right]$$

$$,\ \text{at } x = 0 \Rightarrow \Delta = 0 \ ,\ \text{at } x = \frac{L}{2} \Rightarrow \theta = 0$$

$$\frac{P}{4}\left(\frac{L}{2}\right)^2 + C_1 = 0 \Rightarrow C_1 = -\frac{PL^2}{16}$$

$$\theta = \frac{1}{EI}\left(\frac{Px^2}{4} - \frac{PL^2}{16}\right) \Rightarrow \theta_A = -\frac{PL^2}{16}$$

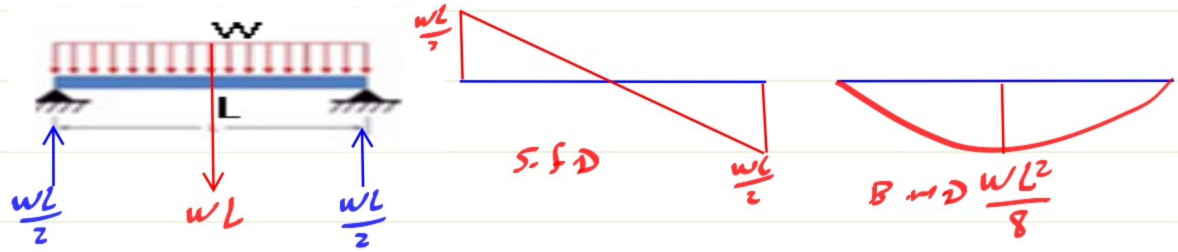
$$,\ \Delta = \frac{P}{12}(0)^3 + C_1(0) + C_2 = 0 \Rightarrow C_2 = 0$$

$$\Delta = \frac{1}{EI}\left(\frac{Px^3}{12} - \frac{PL^2}{16}\cdot x\right)$$

$$\Delta_C = \frac{1}{EI}\left[\frac{P}{12}\left(\frac{L}{2}\right)^3 - \frac{PL^2}{16}\cdot\frac{L}{2}\right] = -\frac{PL^3}{48EI}$$

$$M = \frac{wL}{2} x - \frac{wx^2}{2}$$

(b)



$$\theta = \frac{1}{EI}\int M\,dx = \frac{1}{EI}\left[\frac{wL}{2}\cdot\frac{x^2}{2} - \frac{w}{2}\cdot\frac{x^3}{3} + C_1\right]$$

$$,\ \Delta = \frac{1}{EI}\int \theta\,dx = \frac{1}{EI}\left[\frac{wL}{2}\cdot\frac{x^3}{6} - \frac{w}{2}\cdot\frac{x^4}{12} + C_1 x + C_2\right]$$

$$,\ \text{at } x = 0 \Rightarrow \Delta = 0\ ,\ \text{at } x = \frac{L}{2} \Rightarrow \theta = 0$$

$$\theta = \frac{wL}{4}\left(\frac{L}{2}\right)^2 - \frac{w}{6}\left(\frac{L}{2}\right)^3 + C_1 = 0 \Rightarrow C_1 = -\frac{wL^3}{24}$$

$$\theta = \frac{wL}{4}x^2 - \frac{w}{6}x^3 - \frac{wL^3}{24}$$

$$\text{at } x = 0 \Rightarrow \theta = -\frac{wL^3}{24}$$

,

$$\Delta = \frac{wL}{2}(0) - \frac{w}{2}(0) + C_1(0) + C_2 = 0 \Rightarrow C_2 = 0$$

$$\Delta = \frac{1}{EI}\left[\frac{wL}{12}x^3 - \frac{w}{24}x^4 - \frac{wL^3}{24}x\right]$$

$$\text{at } x = \frac{L}{2}$$

$$\Delta = \frac{1}{EI}\left[\frac{wL}{12}\left(\frac{L}{2}\right)^3 - \frac{w}{24}\left(\frac{L}{2}\right)^4 - \frac{wL^3}{24}\left(\frac{L}{2}\right)\right] = -\frac{5wL4}{384EI}$$

$$,\ \text{maximum binding} = \frac{wL^2}{8}$$

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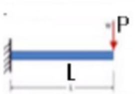
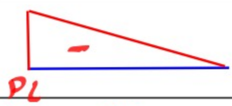
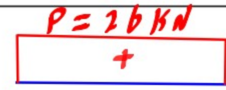
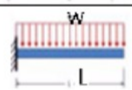
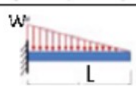
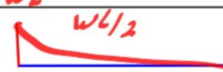
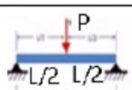
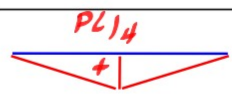
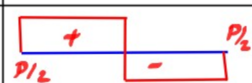
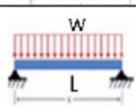
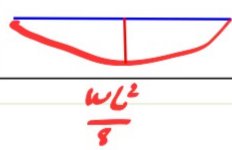
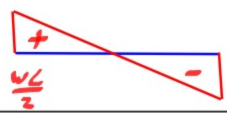
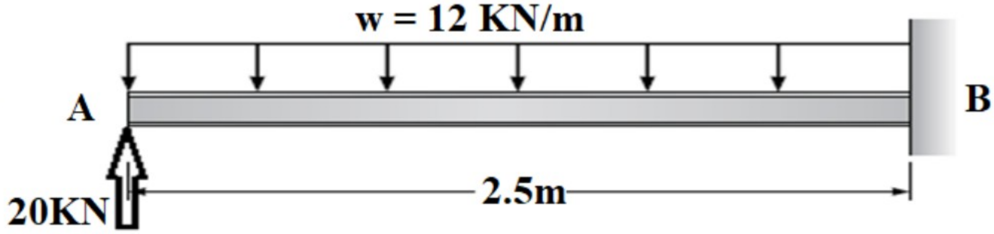
Assignment No (2)

Deflection of Beams - Geometric Methods

1. Complete the table below, take M= 14KN.m, L=2.1m, w=15KN/m and P= 26KN

No	Type of beam	Bending moment diagram	Shear force diagram	Value of Slope	Maximum Value of Deflection
1		 14KN.m −	 0	$\theta = \frac{-Ml}{EI} = \frac{-29.4}{EI}$	$\Delta = \frac{-Ml^2}{2EI} = \frac{-30.87}{EI}$
2		 − PL	 P = 26 KN +	$\theta = \frac{-Pl^2}{2EI} = \frac{-57.33}{EI}$	$\Delta = \frac{-Pl^3}{3EI} = \frac{-80.26}{EI}$
3		 $wL^2/2$ −	 w = 15 KN/m + wL	$\theta = \frac{-wl^3}{6EI} = \frac{-23.15}{EI}$	$\Delta = \frac{-wl^4}{8EI} = \frac{-36.46}{EI}$
4		 $wL^2/6$	 $wL/2$	$\theta = \frac{-wl^3}{24EI} = \frac{-5.78}{EI}$	$\Delta = \frac{-wl^4}{30EI} = \frac{-9.72}{EI}$
5		 $PL/4$ +	 + − P/2 P/2	$\theta = \frac{-Pl^2}{16EI} = \frac{-7.16}{EI}$	$\Delta = \frac{-Pl^3}{48EI} = \frac{-5}{EI}$
6		 $\frac{wL^2}{8}$	 + − $\frac{wL}{2}$	$\theta = \frac{-wl^3}{24EI} = \frac{-5.78}{EI}$	$\Delta = \frac{-5wl^4}{384EI} = \frac{-3.8}{EI}$

2.A propped cantilever beam carries a uniformly distributed load of 20KN/m over its entire length, as shown in figure below. Using the geometric methods, determine the slopes at A,B and the deflection at A.



$$\theta_A = \frac{wl^3}{6EI} - \frac{Pl^2}{2EI} = \frac{12 * 2.5^3}{6EI} - \frac{20 * 2.5^2}{2EI} = \frac{-125}{4EI}$$

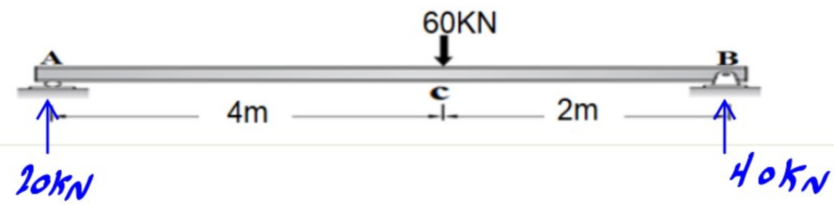
θ_B = zero

$$\Delta_A = \frac{Pl^3}{3EI} - \frac{wl^4}{8EI} = \frac{20 * 2.5^3}{3EI} - \frac{12 * 2.5^4}{8EI} = \frac{45.57}{EI}$$

3.For the steel beam in figure below,compute the end slope at supports A & B. Also determine the location and value of the maximum deflection.

$\Delta_A = 0$

$\Delta_B = 0$



$$M = 20x - 60(x-4)$$

$$\theta = \frac{1}{EI}\int M\,dx = \frac{1}{EI}\left[10x^2 - 30(x-4)^2 + C_1\right]$$

$$\Delta = \frac{1}{EI}\int \theta\,dx = \frac{1}{EI}\left[\frac{10}{3}x^3 - 10(x-4)^3 + C_1 x + C_2\right]$$

at $x = 0 \Rightarrow \Delta = 0 \Rightarrow C_2 = 0$

at $x = 6 \Rightarrow \Delta = 0$

$$0 = \frac{10}{3} * 6^3 - 10(2)^3 + C_1 * 6$$

$$C_1 = -106.67$$

$$\theta = \frac{1}{EI}\left(10x^2 - 30(x-4)^2 - 106.67\right)$$

$$\Delta = \frac{1}{EI}\left(\frac{10}{3}x^3 - 10(x-4)^3 - 106.67x\right)$$

$$\theta_A = \frac{-106.67}{EI}$$

$$\theta_B = \frac{1}{EI}\left(10 * 6^2 - 30(2)^2 - 106.67\right) = \frac{133}{EI}$$

(for max deflection

$$10x^2 - 106.67 = 0 \Rightarrow x = 3.26\,m$$

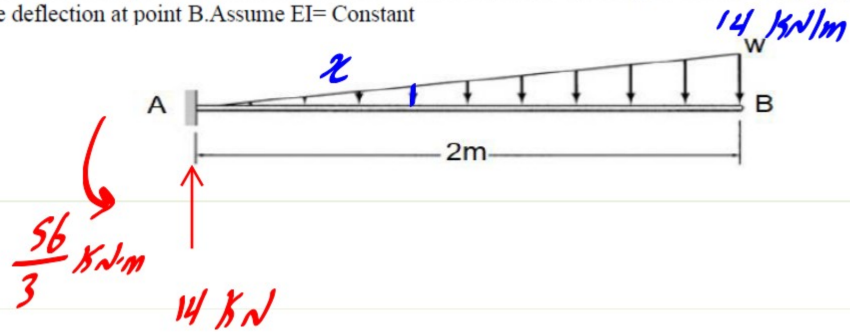
$$\Delta_{max} = \frac{1}{EI}\left(\frac{10}{3} * 3.26^3 - 106.67 * 3.26\right)$$

$$= \frac{-232.26}{EI}$$

4. The RC Cantilever beam is subjected to distributed load w= 14KN/m, as shown in figure below. Using the geometric method, find the slope at A,B and the deflection at point B.Assume EI= Constant

$\frac{14}{2} = \frac{l}{x}$

$l = 7x$



$$M = -\frac{56}{3} + 14x - 0.5x * 7x * \frac{1}{3}x$$

$$M = -\frac{56}{3} + 14x - \frac{7}{6}x^3$$

$$\theta = \int M\,dx = \frac{1}{EI}\left(-\frac{56}{3}x + 7x^2 - \frac{7}{24}x^4 + C_1\right)$$

at $x = 0 \Rightarrow \theta = 0 \Rightarrow C_1 = 0$

$$\theta = -\frac{56}{3}x + 7x^2 - \frac{7}{24}x^4$$

$$\theta_B = \frac{-14}{EI}$$

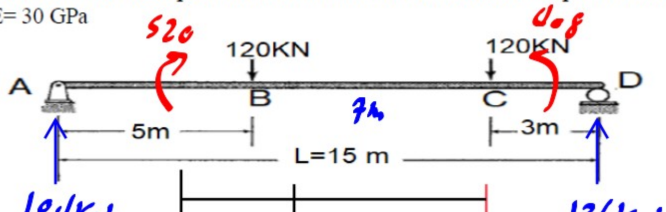
$$\Delta = \frac{1}{EI}\int \theta\,dx = \frac{1}{EI}\left\{-\frac{28}{3}x^2 + \frac{7}{3}x^3 - \frac{7x^5}{120} + C_2\right\}$$

at $x = 0 \Rightarrow \Delta = 0 \Rightarrow C_2 = 0$

$$\Delta_B = -\frac{28}{3} * 2^2 + \frac{7}{3} * 2^3 - \frac{7 * 2^5}{120} = -\frac{20.54}{EI}$$

$$\frac{A}{EI} = \theta_A - \theta_D$$

5.Use the moment-area method to determine the slopes at ends A and D and the deflections at points B and C of the beam shown in Fig. below. Take EI = constant and E= 30 GPa

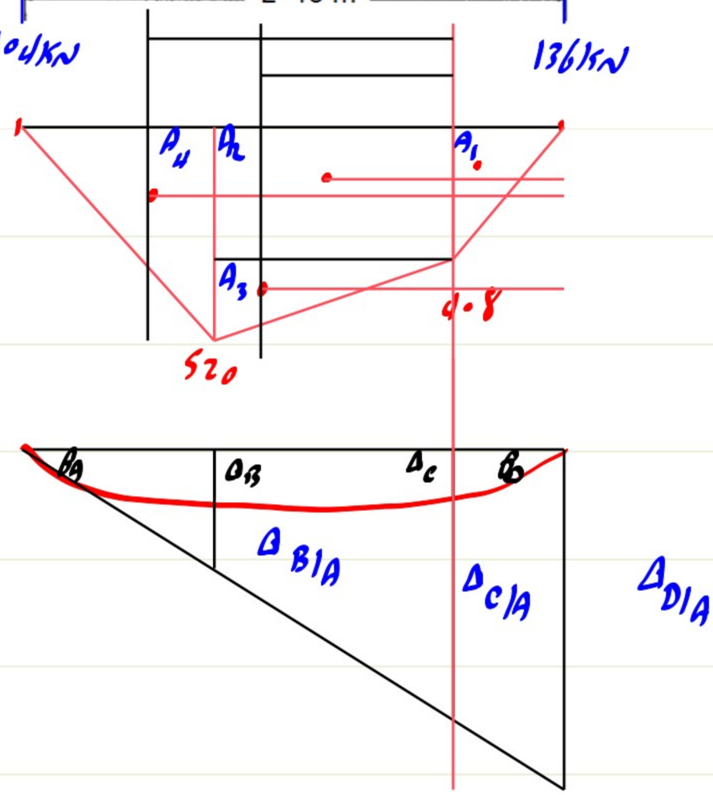


$A_1 = 612$

$A_2 = 2856$

$A_3 = 392$

$A_4 = 1300$



$$\theta_A = \frac{\Delta_{D/A}}{15}$$

$$\Delta_{D/A} EI = 612 * 2 + 2856 * 6.5 + 392 * 7.67 + 1300 * 11.67$$

$$\Delta_{D/A} = \frac{37960}{EI}$$

$$\theta_A = \frac{37960}{15EI} = \frac{2530.67}{30 * 10^6 I} = \frac{8.43 * 10^{-5}}{I}$$

$$\frac{\Delta_{D/A}}{15} = \frac{\Delta_B + \Delta_{B/A}}{5}$$

$\Delta_{B/A} = 1300 * \frac{5}{3} = 2166.67$

$\frac{37960}{15} = \frac{\Delta B + 2166.67}{5}$

$\Delta_B = \frac{10486}{30 * 106}$

$\frac{\Delta_{D/A}}{15} = \frac{\Delta_C + \Delta_{C/A}}{12}$

$EI\Delta_{C/A} = 2856 * 3.5 + 392 * \frac{2}{3} * 7 + 1300 * 8.67$

$\Delta_{C/A} = \frac{23356}{EI}$

$\frac{37960}{15} = \frac{\Delta_C + 23096.6}{12}$

$\Delta_C = \frac{7271}{EI}$

$\theta_A - \theta_D = \frac{A}{EI}$

$\frac{2530.67 - (612 + 2856 + 392 + 1300)}{EI} = \theta_D$

$\theta_D = \frac{-2629.33}{EI}$