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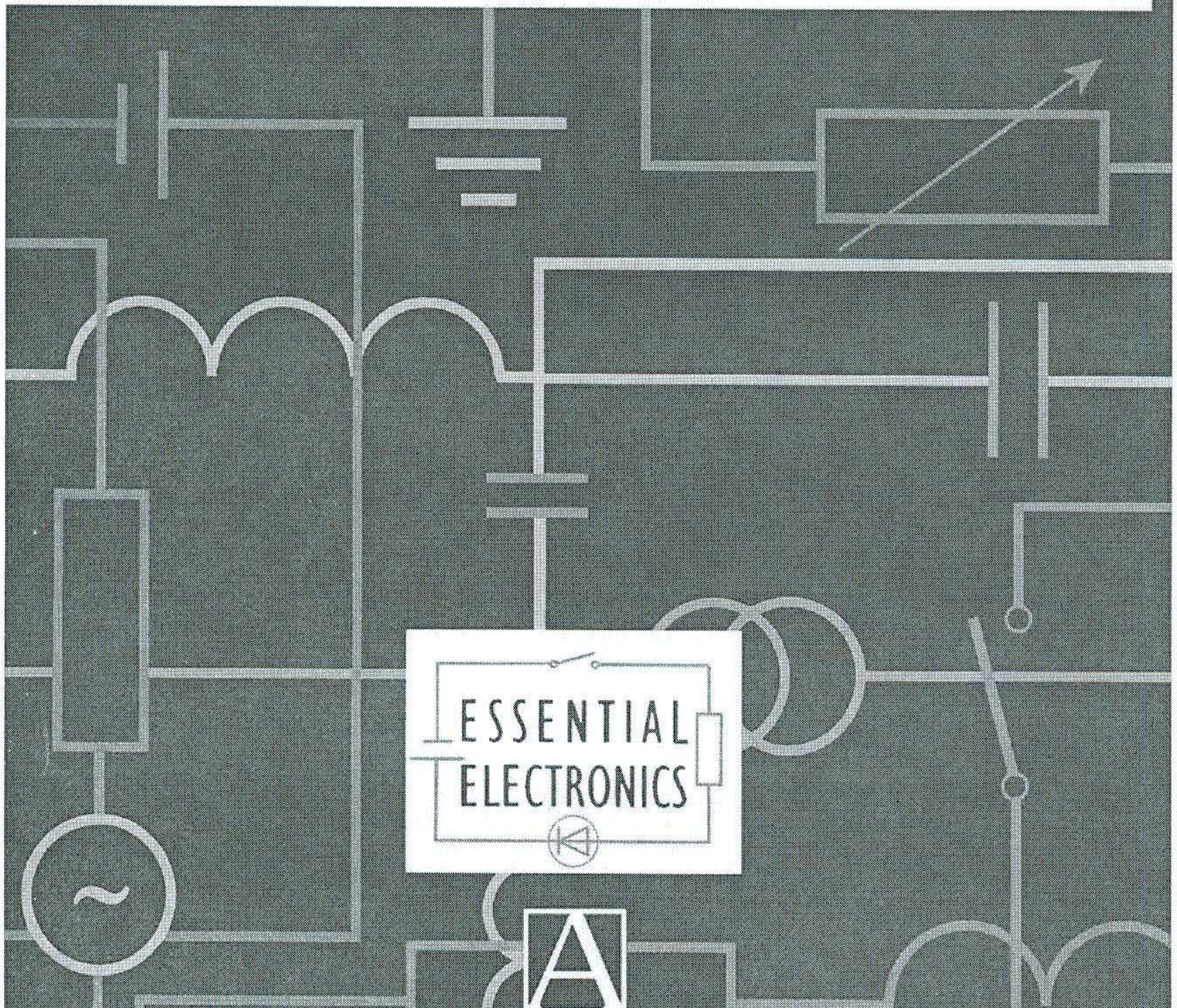
INTRODUCTION TO ELECTRIC CIRCUITS

Ray Powell



ESSENTIAL ELECTRONICS

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To the memory of my mother and father with grateful thanks

Essential Electronics Series

Introduction to Electric Circuits

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Series Preface

In recent years there have been many changes in the structure of undergraduate courses in engineering and the process is continuing. With the advent of modularization, semesterization and the move towards student-centred learning as class contact time is reduced, students and teachers alike are having to adjust to new methods of learning and teaching.

Essential Electronics is a series of textbooks intended for use by students on degree and diploma level courses in electrical and electronic engineering and related courses such as manufacturing, mechanical, civil and general engineering. Each text is complete in itself and is complementary to other books in the series.

A feature of these books is the acknowledgement of the new culture outlined above and of the fact that students entering higher education are now, through no fault of their own, less well equipped in mathematics and physics than students of ten or even five years ago. With numerous worked examples throughout, and further problems with answers at the end of each chapter, the texts are ideal for directed and independent learning.

The early books in the series cover topics normally found in the first and second year curricula and assume virtually no previous knowledge, with mathematics being kept to a minimum. Later ones are intended for study at final year level.

The authors are all highly qualified chartered engineers with wide experience in higher education and in industry.

R G Powell
Jan 1995
Nottingham Trent University

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Preface

This book covers the material normally found in first and second year syllabuses on the topic of electric circuits. It is intended for use by degree and diploma students in electrical and electronic engineering and in the associated areas of integrated, manufacturing and mechanical engineering.

The two most important areas of study for all electrical and electronic engineering students are those of circuit theory and electromagnetic field theory. These lay the foundation for the understanding of the rest of the subjects which make up a coherent course and they are intimately related. Texts on one of them invariably and inevitably have references to the other. In Chapter 2 of this book the ingredients of electric circuits are introduced and the circuit elements having properties called capacitance and inductance are associated with electric and magnetic fields respectively. Faraday's law is important in the concept of mutual inductance and its effects. Reference is made, therefore, to electromagnetic field theory on a need to know basis, some formulae being presented without proof.

The level of mathematics required here has been kept to a realistic minimum. Some facility with algebra (transposition of formulae) and knowledge of basic trigonometry and elementary differentiation and integration is assumed. I have included well over a hundred worked examples within the text and a similar number of problems with answers. At the end of each chapter there is a series of self-assessment test questions.

Ray Powell
Nottingham, November 1994

1 Units and dimensions

1.1 INTRODUCTION

In electrical and electronic engineering, as in all branches of science and engineering, measurement is fundamentally important and two interconnected concepts are involved. First we need to know what it is that we wish to measure, and this is called a quantity. It may be a force or a current or a length (of a line say). The quantity must then be given a unit which indicates its magnitude, that is, it gives a measure of how strong the force is or how big the current is or how long the line is. In any system of units a certain number of physical quantities are arbitrarily chosen as the basic units and all other units are derived from these.

1.2 The *SYSTÈME INTERNATIONAL D'UNITÉS*

This system of units, abbreviated to 'the SI', is now in general use and in this system seven basic quantities, called dimensions, are selected. These are mass, length, time, electric current, thermodynamic temperature, luminous intensity and amount of substance, the first four of which are of particular importance to us in this book. In addition to these seven basic quantities there are two supplementary ones, namely plane angle and solid angle. All of these are shown, together with their unit names, in Table 1.1. These units are defined as follows:

kilogram (kg): the mass of an actual piece of metal (platinum–iridium) kept under controlled conditions at the international bureau of weights and measures in Paris

metre (m): the length equal to 1 650 763.73 wavelengths *in vacuo* of the radiation corresponding to the transition between the levels $2p_{10}$ and $5d_5$ of the krypton-86 atom

second (s): the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium-133 atom

ampere (A): that constant current which, when maintained in each of two

Table 1.1

Quantity	*Unit*	*Unit abbreviation*
Mass	kilogram	kg
Length	metre	m
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol
Plane angle	radian	rad
Solid angle	steradian	sr

infinitely long parallel conductors of negligible cross-sectional area separated by a distance of 1 m in a vacuum, produces a mutual force between them of 2×10^{-7} N per metre length

kelvin (K): the fraction 1/273.16 of the thermodynamic temperature of the triple point of water

candela (cd): the luminous intensity, in the perpendicular direction, of a surface of area 1/600 000 m^2 of a black body at the temperature of freezing platinum under a pressure of 101 325 Pa

mole (mol): the amount of substance of a system which contains as many specified elementary particles (i.e. electrons, atoms, etc.) as there are atoms in 0.012 kg of carbon-12

radian (rad): the plane angle between two radii of a circle which cut off on the circumference an arc equal to the radius

steradian (sr): that solid angle which, having its vertex at the centre of a sphere, cuts off an area of the surface of the sphere equal to that of a square with sides equal to the radius.

The scale temperature (degree Celsius) is the thermodynamic temperature minus 273.16, so that 0 °C corresponds to 273.16 K and 0 K corresponds to −273.16 °C. Note that we write 0 K and 273 K, not 0 °K nor 273 °K.

As an example of how the other units may be derived from the basic units, velocity is length divided by time. It is usual to write these dimensional equations using square brackets, with the basic quantities being in capital letters and the derived quantities being in lower case letters. Thus for this example we can write [v] = [L]/[T], or

$$[\mathrm{v}] = [\mathrm{L\,T^{-1}}] \qquad (1.1)$$

Example 1.1

Obtain the dimensions of (1) acceleration, (2) force, (3) torque.

Solution

$a = \frac{v}{t} = \frac{\frac{L}{T}}{T} = \frac{L}{T^2} = L \cdot T^{-2}$ $m/s^2 = m s^{-2}$

1 Acceleration is the rate of change of velocity, so is velocity/time. Thus

$[a] = [v]/[T]$. From Equation (1.1) we have that $[v] = [L\,T^{-1}]$, so

$$[a] = [L\,T^{-1}]/[T] = [L\,T^{-1}][T^{-1}]$$

or

$$[a] = [L\,T^{-2}] \tag{1.2}$$

M $L\,T^{-2}$

2 Force is mass times acceleration. Thus $[f] = [M][a]$. From Equation (1.2) we have that $[a] = [L\,T^{-2}]$, so

$$[f] = [M][L\,T^{-2}]$$

or

$$[f] = [M\,L\,T^{-2}] \tag{1.3}$$

L

3 Torque is force times the length of the torque arm. Thus $[t] = [f][L]$. From Equation (1.3) we have that $[f] = [M\,L\,T^{-2}]$, so

$$[t] = [M\,L\,T^{-2}][L]$$

or

$$[t] = [M\,L^2\,T^{-2}] \tag{1.4}$$

Example 1.2

Determine the dimensions of (1) energy, (2) power.

Solution

$M\,L\,T^{-2}$ L

1 Energy is work, which is force multiplied by distance. Thus $[w] = [f][L]$. From Equation (1.3) we have that $[f] = [M\,L\,T^{-2}]$, so

$$[w] = [M\,L\,T^{-2}][L]$$

or

$$[w] = [M\,L^2\,T^{-2}] \tag{1.5}$$

2 Power is energy divided by time. Thus $[p] = [w]/[T]$. From Equation (1.5) we have that $[w] = [M\,L^2\,T^{-2}]$, so

$$[p] = [M\,L^2\,T^{-2}]/[T] = [M\,L^2\,T^{-2}][T^{-1}]$$

or

$$[p] = [M\,L^2\,T^{-3}] \tag{1.6}$$

Example 1.3

Find the dimensions of (1) electric charge, (2) electric potential difference.

Solution

1 Electric charge is electric current multiplied by time. Thus [q] = [A][T], so

[q] = [A T] $\frac{w}{q} = \frac{M L^2 T^{-3}}{A \cancel{T}} =$ (1.7)

2 When a charge of 1 coulomb is moved through a potential difference of 1 volt the work done is 1 joule of energy, so that electric potential difference is energy divided by electric charge. Thus [pd] = [w]/[q]. From Equation (1.5) we have that [w] = $[\mathrm{M\,L^2\,T^{-2}}]$, and from Equation (1.7) we see that [q] = [A T], so

$$[\mathrm{pd}] = [\mathrm{M\,L^2\,T^{-2}}]/[\mathrm{A\,T}] = [\mathrm{M\,L^2\,T^{-2}}][\mathrm{A^{-1}\,T^{-1}}]$$

or

$$[\mathrm{pd}] = [\mathrm{M\,L^2\,T^{-3}\,A^{-1}}] \tag{1.8}$$

Example 1.4

Obtain the dimensions of (1) resistance, (2) inductance, (3) capacitance.

Solution

1 Resistance is electric potential difference divided by electric current. From Equation (1.8) the dimensions of electric potential difference are $[\mathrm{M\,L^2\,T^{-3}\,A^{-1}}]$. Thus [r] = $[\mathrm{M\,L^2\,T^{-3}\,A^{-1}}]/[\mathrm{A}]$, so

$$[\mathrm{r}] = [\mathrm{M\,L^2\,T^{-3}\,A^{-2}}] \tag{1.9}$$

2 The magnitude of the emf induced in a coil of inductance L when the current through it changes at the rate of I ampere in t seconds is given by

$l = \frac{e \cdot t}{I}$

$e = LI/t$, where e is measured in volts and is a potential difference. Thus the dimensions of L are given by [l] = [pd][T]/[A]. From Equation (1.8), [pd] = $[\mathrm{M\,L^2\,T^{-3}\,A^{-1}}]$, so

$$[\mathrm{l}] = [\mathrm{M\,L^2\,T^{-2}\,A^{-2}}] \tag{1.10}$$

3 Capacitance (C) is electric charge (Q) divided by electric potential difference (V). From Equation (1.7), [q] = [A T]. From Equation (1.8), [pd] = $[\mathrm{M\,L^2\,T^{-3}\,A^{-1}}]$. Thus [c] = $[\mathrm{A\,T}]/[\mathrm{M\,L^2\,T^{-3}\,A^{-1}}]$, so

$$[\mathrm{c}] = [\mathrm{M^{-1}\,L^{-2}\,T^4\,A^2}] \tag{1.11}$$

1.3 DIMENSIONAL ANALYSIS

A necessary condition for the correctness of an equation is that it should be dimensionally balanced. It can be useful to perform a dimensional analysis on equations to check their correctness in this respect. This can be done by checking that the dimensions of each side of an equation are the same.

Example 1.5

The force between two charges q_1 and q_2 separated by a distance d in a vacuum is given by $F = q_1 q_2 / 4\pi\epsilon_0 d^2$, where ϵ_0 is a constant whose dimensions are $[\mathrm{M}^{-1}\,\mathrm{L}^{-3}\,\mathrm{T}^{4}\,\mathrm{A}^{2}]$. Check the dimensional balance of this equation.

Solution

The left-hand side of the equation is simply the force F and from Example 1.1 (2) we see that its dimensions are $[\mathrm{M\,L\,T}^{-2}]$.

The dimensions of the right-hand side are $[q][q]/[4][\pi][\epsilon_0][d^2]$. From Example 1.3 (1) we see that the dimensions of electric charge $[q]$ are $[\mathrm{A\,T}]$. Numbers are dimensionless so that the figure 4 and the constant π have no dimensions. We are told in the question that the dimensions of ϵ_0 are $[\mathrm{M}^{-1}\,\mathrm{L}^{-3}\,\mathrm{T}^{4}\,\mathrm{A}^{2}]$. The distance between the charges, d, has the dimensions of length so that d^2 has the dimensions $[L^2]$. The dimensions of the right-hand side of the equation are therefore

$$[\mathrm{A\,T}][\mathrm{A\,T}]/[\mathrm{M}^{-1}\,\mathrm{L}^{-3}\,\mathrm{T}^{4}\,\mathrm{A}^{2}][\mathrm{L}^2] = [\mathrm{A}^2\,\mathrm{T}^2][\mathrm{M\,L}^3\,\mathrm{T}^{-4}\,\mathrm{A}^{-2}\,\mathrm{L}^{-2}] = [\mathrm{M\,L\,T}^{-2}]$$

which is the same as that obtained for the left-hand side of the equation. The equation is therefore dimensionally balanced.

Example 1.6

The energy in joules stored in a capacitor is given by the expression $(C^a V^b)/2$, where C is the capacitance of the capacitor in farads and V is the potential difference in volts maintained across its plates. Use dimensional analysis to determine the values of a and b.

$W = \dfrac{C^a V^b}{2}$

Solution

We have that $W = (C^a V^b)/2$
In dimensional terms $[w] = [c]^a[pd]^b$
From Equation (1.5), $[w] = [\mathrm{M\,L}^2\,\mathrm{T}^{-2}]$
From Equation (1.11), $[c] = [\mathrm{M}^{-1}\,\mathrm{L}^{-2}\,\mathrm{T}^4\,\mathrm{A}^2]$
From Equation (1.8), $[pd] = [\mathrm{M\,L}^2\,\mathrm{T}^{-3}\,\mathrm{A}^{-1}]$ therefore

$$[\mathrm{M\,L}^2\,\mathrm{T}^{-2}] = [\mathrm{M}^{-1}\,\mathrm{L}^{-2}\,\mathrm{T}^4\,\mathrm{A}^2]^a[\mathrm{M\,L}^2\,\mathrm{T}^{-3}\,\mathrm{A}^{-1}]^b$$

Equating powers of [M], $1 = -a + b \Rightarrow b = a + 1$
Equating powers of [A], $0 = 2a - b \Rightarrow b = 2a$
By substitution, $2a = a + 1$, so

$W = \dfrac{C^1 V^2}{2} = \dfrac{1}{2} C V^2$

$a = 1$ and $b = 2a = 2$

The values required are therefore $a = 1$; $b = 2$

1.4 MULTIPLES AND SUBMULTIPLES OF UNITS

1μV 100kV

There is an enormous range of magnitudes in the quantities encountered in electrical and electronic engineering. For example, electric potential can be lower than 0.000 001 V or higher than 100 000 V. By the use of multiples and submultiples we can avoid having to write so many zeros. Table 1.2 shows their names and abbreviations.

Table 1.2

Multiple	*Abbreviation*	*Value*	*Submultiple*	*Abbreviation*	*Value*
exa	E	10^{18}	milli	m	10^{-3}
peta	P	10^{15}	micro	μ	10^{-6}
tera	T	10^{12}	nano	n	10^{-9}
giga	G	10^{9}	pico	p	10^{-12}
mega	M	10^{6}	femto	f	10^{-15}
kilo	k	10^{3}	atto	a	10^{-18}

These are the preferred multiples and submultiples and you will see that the powers are in steps of 3. However, because of their convenience there are some others in common use. For example, deci (d), which is 10^{-1}, is used in decibel (dB); and centi (c), which is 10^{-2}, is used in centimetre (cm). Capital letters are used for the abbreviations of multiples and lower case letters are used for the abbreviations of submultiples. The exception is kilo for which the abbreviation is the lower case k, not the capital K.

Example 1.7

10^{-3} 10^{-6}

Express 10 seconds in (1) milliseconds, (2) microseconds. 10×10^6

Solution

$\frac{1s}{10^{-3}} = 10 \times 10^3 = 10^4$ ms

1 To convert from units to multiples or submultiples of units it is necessary to *divide* by the multiple or submultiple. To find the number of milliseconds in 1 second we simply divide by the submultiple 10^{-3}. Thus 1 second = $1/10^{-3} = 10^3$ milliseconds. In 10 seconds there are therefore $10 \times 10^3 = 10^4$ ms.

2 To find the number of microseconds in 10 seconds we divide by the submultiple 10^{-6}. Thus in 10 s there are $10/10^{-6} = 10^7$ μs.

Example 1.8

10^3 10^{-2}

Express 1 metre in (1) kilometres, (2) centimetres.

Solution

1 To find the number of kilometres in 1 metre we divide by the multiple 10^3. Thus in 1 m there are $1/10^3 = 10^{-3}$ km.

2 To find the number of centimetres in 1 metre we divide by the submultiple 10^{-2}. Thus in 1 m there are $1/10^{-2} = 10^2$ cm.

Example 1.9

Express (1) 10 mV in volts, (2) 150 kW in watts.

Solution

1 To convert from multiples or submultiples to units it is necessary to *multiply* by the multiple or submultiple. Thus to convert from millivolts to volts we multiply by the submultiple 10^{-3}:

$$10 \text{ mV} = 10 \times 10^{-3} \text{ V} = 10^{-2} \text{ V}$$

2 Similarly, to convert from kilowatts to watts we multiply by the multiple 10^3:

$$150 \text{ kW} = 150 \times 10^3 \text{ W} = 1.5 \times 10^5 \text{ W}$$

Example 1.10

Express (1) 10 mV in MV, (2) 5 km in mm, (3) 0.1 μF in pF, (4) 50 MW in GW.

$\frac{10^{-2} V}{10^6} = 10^{-2} \times 10^{-6}$, $\frac{5 \times 10^3}{10^{-3}} = 5 \times 10^6$ mm , $\frac{0.1 \times 10^{-6}}{10^{-12}} = 0.1 \times 10^{-6} \times 10^{12}$

Solution

1 To convert from millivolts to volts we multiply by the submultiple 10^{-3}. Thus

$$10 \text{ mV} = (10 \times 10^{-3}) \text{ V} = 10^{-2} \text{ V}$$

Then to convert to megavolts we divide by the multiple 10^6. Thus

$$10^{-2} \text{ V} = (10^{-2}/10^6) \text{ MV} = 10^{-2} \times 10^{-6} \text{ MV} = 10^{-8} \text{ MV}$$

Therefore, in 10 mV there are 10^{-8} MV

2 In this case we multiply by the multiple kilo (10^3) and then divide by the submultiple milli (10^{-3}):

$$5 \text{ km} = 5 \times 10^3 \text{ m} = (5 \times 10^3/10^{-3}) \text{ mm} = 5 \times 10^6 \text{ mm}$$

3 First multiply by the submultiple micro (10^{-6}) and then divide by the submultiple pico (10^{-12}):

$$0.1 \text{ μF} = 0.1 \times 10^{-6} \text{ F} = 10^{-7} \text{ F} = (10^{-7}/10^{-12}) \text{ pF} = 10^5 \text{ pF}$$

4 Here we multiply by the multiple mega (10^6) and then divide by the multiple giga (10^9):

$$50 \text{ MW} = 50 \times 10^6 \text{ W} = 5 \times 10^7 \text{ W} = (5 \times 10^7/10^9) \text{ GW} = 5 \times 10^{-2} \text{ GW}$$

1.5 SELF-ASSESSMENT TEST

1 Which of the following are units:
torque; second; newton; time; kilogram?

2 Which of the following are derived units:
metre; coulomb; newton; ampere; volt?

3 Obtain the dimensions of magnetic flux (ϕ) given that the emf induced in a coil of N turns in which the flux is changing at the rate of ϕ webers in t seconds is $N\phi/t$ volts.

4 Obtain the dimensions of magnetic flux density, B (magnetic flux per unit area).

5 Determine the dimensions of potential gradient (change in potential with distance ($\mathrm{d}V/\mathrm{d}x$))

6 Express:
(a) 30 mA in amperes
(b) 25 A in microamperes
(c) 10 MW in milliwatts
(d) 25 nC in coulombs
(e) 150 pF in nanofarads
(f) 60 MW in gigawatts
(g) 150 μJ in millijoules
(h) 220 Ω in kilohms
(i) 55 MΩ in milliohms
(j) 100 N in kilonewtons.

1.6 PROBLEMS

1 Determine the dimensions of magnetic field strength (H) which is measured in amperes per metre.

2 The permeability (μ) of a magnetic medium is the ratio of magnetic flux density (B) to magnetic field strength (H). Determine its dimensions.

3 Obtain the dimensions of electric flux which is measured in coulombs.

4 Electric flux density (D) is electric flux per unit area. Obtain its dimensions.

5 Find the dimensions of electric field strength whose unit is the volt per metre.

6 Check the validity of the statement that the volt per metre is equivalent to the newton per coulomb.

7 The permittivity (ϵ) of the medium of an electric field is the ratio of electric flux density (D) to electric field strength (E). Find its dimensions.

8 The power in watts dissipated in a resistance R ohms through which is flowing a current of I amperes is given by $P = I^a R^b$. Use dimensional analysis to obtain the values of a and b.

9 'The energy (in joules) stored in an inductor having an inductance of L henry through which a current of I amperes is flowing is given by $W = (L^2 I)/2$.' Check whether this statement is true using dimensional analysis.

10 The maximum torque of a three-phase induction motor is given by $T_{\text{max}} = kE_2^a X_2^b \omega^c$ where k is a constant, E_2 is the rotor-induced emf measured in volts, X_2 is the rotor reactance measured in ohms and ω is the angular frequency measured in radians per second. Determine the values of a, b and c.

11 The force between two similar magnetic poles of strength p webers separated by a distance d metres in a medium whose permeability is μ is given by $F = kp^a \mu^b d^c$. Obtain the values of a, b and c.

12 Given that the power (in watts) is the product of potential difference (in volts) and current (in amperes), obtain a value for the power in megawatts (MW) dissipated in a resistance when the current through it is 0.35 kA and the potential difference across it is 4.15×10^8 μV.